

The quantum measurement problem as a witness to “It from bit”

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A conceptual difficulty in the foundations of quantum mechanics is the quantum measurement problem (QMP), essentially concerned with the apparent non-unitarity of the measurement process and the classicality of macroscopic systems. In an information theoretic approach proposed by us earlier (Quantum Information Processing 2, 153, 2003), which we clarify and elaborate here, QMP is understood to signal a fundamental finite resolution of quantum states, or, equivalently, a discreteness of Hilbert space. This was motivated by the notion that physical reality is a manifestation of information stored and discrete computations performed at a deeper, sub-physical layer. This model entails that states of sufficiently complex, entangled systems will be unresolvable, or, *computationally unstable*. Wavefunction collapse is postulated as an error preventive response to such computational instability. In effect, sufficiently complex systems turn classical because of the finiteness of the computational resources available to the physical universe. We show that this model forms a reasonable complement to decoherence for resolving QMP, both in respect of the problem of definite outcomes and of the preferred basis problem. The model suggests that QMP, as a window on the sub-physical universe, serves as a witness to Wheeler’s koan “it from bit”. Some implications for quantum computation and quantum gravity are examined.

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I. INTRODUCTION

The quantum measurement problem (QMP) is a key set of questions that, arguably, every interpretation of quantum mechanics must answer. The principal problem is that the wavefunction in quantum mechanics evolves unitarily according to the Schrödinger equation into a linear superposition of different states but measurements always find the physical system in a definite state, typically a position eigenstate. The future evolution is based on the system having the measured value at that point in time, meaning that measurement seems to affect the system in a way not explained by the basic theory. Formally, measurement precipitates or collapses the system irreversibly, probabilistically and non-unitarily into a definite state.

In spite of the fundamental nature of QMP, there appears to be no unanimous agreement as regards its resolution, or even significance. Various interpretations or models of quantum measurement have been proposed to resolve it. The early Copenhagen interpretation averred that only discussion about probabilities was meaningful. Rooted in philosophical positivism, this view regarded the wavefunction as a mathematical tool requiring no deeper explanation. The Copenhagen interpretation appears to be more of a way to talk about quantum mechanical “weirdness” in classical language, rather than a true interpretation or model. In particular, its stark contrast of classical, measuring systems from the measured, quantum systems arguably only shifts QMP to a different base, without really resolving it.

Now it is known that an inevitable component of the measurement process is decoherence, by which a system loses coherence through entanglement with the measuring apparatus and the environment [1, 2, 3, 4]. Decoherence is sometimes regarded as resolving QMP, though it is not clear that this is the case [5]. In a given measurement whose outcome has been read-out and is thus known, the measured system is formally represented by a pure state. Since the reduced density operator for an entangled system is necessarily mixed, a pure state cannot be entangled with any other system, be it the measuring instrument or the unknown environment. This line of reasoning suggests that a decohering procedure like Eq. (1) below, although able to explain non-selective measurements, is unable to account for selective measurements, that is, the selection of a single, definite outcome state [6]. To fully account for measurement, it would seem that decoherence has to be complemented with either a relative-state interpretation [7] or an explicit breakdown of the superposition principle, which is wavefunction collapse [8]. A problem with the former interpretation is that it does not seem to lead to the probabilistic nature of the real-world measurements. A difficulty with collapse models is that it may look ad hoc, if not motivated on deeper grounds.

Detailed models of wavefunction collapse that has been extensively studied. In the dynamical program [9], and the gravitational model by Penrose [10], the projection postulate is derived as a consequence of additional physics. In

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the former, this is achieved via a small stochastic, nonlinear term added to the dynamical equation of the standard theory; in the latter, via an energy uncertainty arising from gravitational field superpositions due to states that are spatially apart. The model we present here is similar in spirit, but with state vector reduction attributed to a certain computational rather than physical or dynamical cause. It aims to resolve QMP within the decoherence scenario. For details of various other approaches to resolve QMP, cf. the introduction in Ref. [6].

In Ref. [6], we were led to the position that wavefunction collapse, in conjunction with decoherence, is a reasonable way to resolve QMP. In this work, we revisit and concretize some of the ideas presented there. To present QMP somewhat more formally: suppose the system to be measured is in the state $|\psi\rangle = \sum_j c_j |j\rangle$. Further suppose the complex of the measuring apparatus and the environment interacts with the system according to: $|j\rangle|R\rangle|E_0\rangle \rightarrow |j\rangle|m_j\rangle|E_j\rangle$, where $\{|R\rangle, |m_j\rangle\}$ is a complete basis for the measuring apparatus, and $\{|E_0\rangle, |E_j\rangle\}$ a complete, decohered basis for the environment. A puzzle posed by QMP is that the system's measurement does not discernably lead to the observation of superposition $|\Psi'\rangle$ obtained according to

$$|\Psi\rangle \equiv \sum_j c_j |j\rangle|R\rangle|E_0\rangle \longrightarrow |\Psi'\rangle \equiv \sum_j c_j |j\rangle|m_j\rangle|E_j\rangle, \quad (1)$$

as linearity suggests, but (as far as the system-apparatus complex is concerned) probabilistically to *one* of the outcomes $|j\rangle|m_j\rangle$. Note that, non-selectively, the procedure (1) leads to the statistical mixture $\rho = \sum_j |c_j|^2 |j\rangle|m_j\rangle\langle j|\langle m_j|$. To say that this replacement of quantum coherence with classical correlations explains the classicality of the measurement outcomes misses the point of QMP!

Two problems are raised by QMP. The first one concerns the absence of coherent superpositions after measurement. This is called the problem of definite outcomes (QMP1). The second problem asks how the choice of particular basis in which the ‘collapse’ happens is made. This is the preferred basis problem (QMP2).

II. THE COMPUTATIONAL MODEL FOR QUANTUM MEASUREMENT (CMQM)

The basic philosophy behind our present model, called the computational model for quantum measurement (CMQM) [6], is that in some sense physical reality is fundamentally informational and computational. We picture the universe as composed of two layers: an apparent, physical superstructure (“physical universe”), and a hidden, algorithmic matrix (“sub-physical universe”) supporting it. The states and evolution of systems in the physical universe correspond to information stored and computations performed by the sub-physical universe. Physical laws correspond to the underlying *discrete* sub-physical computational algorithms.

The idea that physical phenomena emerge from discrete informational or computational processes is not an entirely new notion. Wheeler’s phrase “it from bit” first gave voice to this [11]. Refs. [12] and [13] have proposed ways to obtain physical dynamics from discrete algorithms. The novel feature of our model is that it connects the latter with QMP. In this way, our model suggests that the classicality of the macroscopic world is a window on discrete, sub-physical information processing and that therefore QMP potentially serves as a witness to a profound connection between physical reality and discrete, sub-physical processes.

We begin by positing that quantum states in finite dimension D are *algorithmically bounded*, i.e., that the total information (in bits) required to specify such a state is finite. We identify this information with the Kolmogorov complexity or algorithmic information [14] of the state, in the sense that it is a minimal description of the state with respect to some fiducial basis. This suggests that there exists an intrinsic limit μ to the accuracy with which a quantum state can be specified. We call μ the state resolution parameter. An immediate consequence is that quantum operations are also algorithmically bounded. Algorithmic boundedness is based on the constructivist philosophy that, even though a conventional quantum state living in a Hilbert space is in fact a platonic entity requiring infinite number of bits to be specified, still any physical instance thereof is always finite. This limit is understood to come from basic discreteness of Hilbert space itself, in a sense clarified below. A rather mundane interpretation of the algorithmic information of the quantum state is that it is the amount of memory space at μ -bit accuracy that a sub-physical simulation must allocate for a physical state.

More precisely, we define the algorithmic information for states in dimension D to be the information:

$$\mathcal{A}(D) \equiv - \sum p_j \log p_j = D\mu \text{ bits}, \quad (2)$$

where, for simplicity, we have ignored the fact that because of normalization, $(D-1)\mu$ suffice to specify the state at μ -bit accuracy. Here logarithms refer to base 2 by default. Thus the algorithmic information for a state depends only on its dimension, with μ bits per amplitude (dimension): $\mu/2$ for the real part, and $\mu/2$ for the imaginary part. In contrast, note that the maximum accessible information, $\log D$, is exponentially smaller. Algorithmic boundedness

can be given a geometric interpretation. If we express the distance between states in a finite dimensional Hilbert space in terms of Hilbert space angle, a measure of distance on projective Hilbert space, given by the Fubini-Study metric [15], then the minimum resolvable separation between two distinct states is $2^{-\mu/2}$. In the limit $\mu \rightarrow \infty$, we recover the conventional continuum state space.

It is known that the energy E of the system upper-bounds the speed at which the system can perform *classical* computations, which is roughly E/\hbar logical operations per second (ops) [16]. To see this, for instance consider the evolution of a qubit with logical states $|0\rangle$ and $|1\rangle$ on which we perform the NOT operation. To flip the qubit one can apply a potential $\hat{H}_0 = E_0|E_0\rangle\langle E_0| + E_1|E_1\rangle\langle E_1|$ with energy eigenstates $|E_0\rangle = (1/\sqrt{2})(|0\rangle + |1\rangle)$ and $|E_1\rangle = (1/\sqrt{2})(|0\rangle - |1\rangle)$. Because $|0\rangle = (1/\sqrt{2})(|E_0\rangle + |E_1\rangle)$ and $|1\rangle = (1/\sqrt{2})(|E_0\rangle - |E_1\rangle)$, each logical state has an energy spread $\Delta E = (E_1 - E_0)/2$. Under application of the potential, the system prepared in state $|0\rangle$ after time t becomes: $|\Psi(t)\rangle = \frac{1}{\sqrt{2}}(|E_0\rangle + e^{i2\Delta Et/\hbar}|E_1\rangle)$. From this it follows that the time taken to flip the qubit to $|1\rangle$, and likewise vice-versa, is given by $\pi\hbar/2\Delta E$. One can similarly show that AND and FANOUT gates, and hence a set of gates universal for classical computation, can be accomplished in about similar time. Therefore, the speed of performing classical logical operations is $f_{\text{classical}} \sim 2E/\hbar\pi$ ops, where we set $E_0 = 0$. Suppose n qubits with total energy E process some information. Then each performs operations at rate $\sim 2E/n\hbar\pi$, so that the total rate is $2E/\hbar\pi$, independent of n .

The implication of algorithmic boundedness of operations is that the evolution of any state $|\psi\rangle$ is equivalent to logical operations performed by the sub-physical matrix at the finite rate:

$$\mathcal{F}(D, E_j) = \frac{2^{\mu/2}}{\hbar} \sum_j E_j \equiv \frac{2^{\mu/2} D \bar{E}}{\hbar} \text{ ops.} \quad (3)$$

This may be seen as follows. In the discretized state space, note that the j th amplitude evolves in time $2\pi\hbar/E_j$ through 2π radians. That is, in the complex plane it sweeps through about $2\pi \times 2^{\mu/2}$ cells, so that the rate per amplitude is $\nu_j = 2^{\mu/2} E_j/\hbar$ ops, from which we obtain Eq. (3). The evolution is discrete. Intuitively, one might visualize continuous evolution, where the state vector ‘snaps to’ the nearest lattice cell [19] in discrete time-steps executed at rate ν_j . Any change during time intervals smaller than \mathcal{F}^{-1} is deemed unresolved and undefined. Thus, μ determines both the precision to which states, and their unitary evolution, can be resolved. A rather mundane interpretation of the $\mathcal{F}(D, E_j)$ is that it is the time-step rate in a sub-physical simulation of the evolution of state $|\psi\rangle$ at μ -bit precision.

In Eq. (3), let the E_j ’s be approximately equal, approximated by E . If the system comprises of N qubits, then $D = 2^N$, and the rate at which classical computations are performed is at most about NE/\hbar . On the other hand, both \mathcal{A} and \mathcal{F} scale exponentially with the size of a composite system of particles, in contrast to classical information storage capacity and the maximum classical computational rate. It is simple to see that \mathcal{A} and \mathcal{F} by far exceed the computational power of conventional computers. Suppose we estimate that there are 10^9 computers in all, each with a memory capacity of 10^{12} terabits, a clockrate of 10^9 Hz and 10^5 logical operations per clock cycle. Therefore the combined memory capacity and computational rate of all computers together is 10^{21} bits and 10^{23} ops, respectively. For instance, if $\mu = 50$ bits (a better attempt to estimate μ is discussed later), from Eq. (2) we see that these resources suffice to support the state of no more than 64 qubits, and the computational resources for tracking the evolution of not a single qubit more energetic than a cosmic microwave background photon (temperature $T \approx 2.8$ K).

With the discretization of state space \mathcal{H} , it is not clear that the mathematical structure of the discretized state space so obtained, denoted \mathcal{H}_μ , is strictly a Hilbert space, because the resolution-limited amplitudes do not form a field. One might consider whether \mathcal{H}_μ is a vector space over the finite field of Gaussian integers [18] modulo a very large prime p of order $2^{\mu/2}$? Probably not, because this can easily be shown to lead to a situation where, given integers a, b such that $a < b$, and $\psi \sim a|0\rangle + b|1\rangle$, still $a^2 > b^2 \pmod{p}$, which is inconsistent with the probabilistic interpretation of amplitudes. There may be no simple structure to \mathcal{H}_μ . Still, as a matter of terminology, we will usually call \mathcal{H}_μ as discretized Hilbert space. Also note that states are not in general truly normalized for finite μ . Likewise operations are not strictly unitary, but finite approximations thereof. A unitary operation U in \mathcal{H} is replaced by its μ -bit discretization, the ‘ μ -unitary’ operation U_μ .

One might be concerned about the consistency of such an approximation scheme. One worry might be that $SU(N)$ group structure of the rotation of an N -level system may not be obtained as the limit of ever larger finite discrete subgroups. However, there are models in which continuous symmetry (rotational or Lorentz) is recovered in the long wavelength limit from underlying dynamics with only discrete symmetry, an immediate example being lattice QCD [19]. Another example is that of a model of spinless point particles hopping on a flux lattice, which gives rise to low-energy excitations obeying the Dirac equation [20].

Similar results concerning discreteness can be deduced also for the Schrödinger equation. To see this, we only note that there exist finite simulation algorithms for classical digital computers, which compute discrete valued $\psi(x)$ in discrete space and time steps, and can approximate continuum Schrödinger evolution to arbitrary accuracy. We

therefore regard the μ -limited quantum mechanics (QM_μ) formally as a sub-physical simulation of continuum quantum mechanics (QM), consuming finite computational power and memory, and parametrized by finite constant μ . One then regards conventional \mathcal{H} as the continuum approximation to \mathcal{H}_μ .

A. An entanglement monotone as measure of system complexity

Given a collection of objects, the total algorithmic information \mathcal{A} required to describe it depends on whether the objects are entangled or not. In particular, \mathcal{A} will depend on the combinatorics of the entanglement between the various objects in the collection. For example, the algorithmic information to describe two separable objects of dimensions D_1 and D_2 is $(D_1 + D_2)\mu$ bits. If now the two objects interact to become entangled, then the combined system's updated algorithmic information is $D_1 D_2 \mu$ bits. More generally, if N initially separable D -level systems become entangled, correspondingly \mathcal{A} rises from $DN\mu$ to $D^N \mu$. The question of detecting entanglement in a general multi-partite state is still an open question. Fortunately, we need concern ourselves only with the simpler, pure state entanglement. This is because the sub-physical matrix always 'knows' the state it is simulating, as it were.

It may turn out that the entangled state does not resolvably differ from a separable state, and is thus effectively separable at μ -bit precision. We need a basis-independent way of describing how a state may resolvably differ from a separable state. Note that, given $|\psi\rangle = \sum_{j=0}^{D^N-1} \alpha_j |j\rangle$ that lives in the space of pure states of N D -dimensional objects, which is $\mathbb{C}^D \otimes \cdots \otimes \mathbb{C}^D$, not all of the D^N complex parameters have nonlocal significance. Two states are equivalent modulo local operations as far as their nonlocal properties are concerned if they may be reached from each other by local unitary transformations, given by the group $U(D)^N$, or $U(1) \times SU(D)^N$ if only independent effects are considered. Each equivalence class of nonlocally equivalent states is an orbit of this group. Hence, the space of orbits is $\frac{\mathbb{C}^D \otimes \cdots \otimes \mathbb{C}^D}{U(1) \times SU(D)^N}$ [17]. From this we find that the number of independent nonlocal (real) invariants $\tau(\alpha_j)$ under local unitary rotations must be

$$D^{N+1} - (D^2 - 1)N - 1. \quad (4)$$

A state would be deemed resolvably entangled if the $\tau(\alpha_j)$'s differ sufficiently from their separable values. This does not appear to be a simple prescription for entanglement resolvability in terms of amplitude resolvability, as the $\tau(\alpha_j)$'s can have complicated functional forms. A simpler method is suggested below.

Consider the N -particle state $|\Psi\rangle = \sum_{\alpha_1, \dots, \alpha_N} c_{\alpha_1, \dots, \alpha_N} |\alpha_1, \dots, \alpha_N\rangle$. Let the set of all particles be $T = \{1, 2, 3, \dots, N\}$. Any non-vanishing proper subset of T is denoted y . That is, $y \in \mathcal{T} \equiv 2^T - \emptyset - T$. Denote $\bar{y} \equiv T - y$. Single particle marginal density matrices are denoted ρ_j .

A simple entanglement monotone for an N -partite pure state is:

$$\xi^{(N)} = \begin{cases} \sum_{j=1}^N S(\rho_j) & \text{if } S(\rho_y) \neq 0 \ \forall \ y \in \mathcal{T}. \\ 0 & \text{otherwise,} \end{cases} \quad (5)$$

where $S(\rho_y) = -\text{Tr}[\rho_y \log(\rho_y)]$ is the von Neumann entropy and $\rho_y \equiv \text{Tr}_{\bar{y}}(|\Psi\rangle\langle\Psi|)$. That $\xi^{(N)}$ is indeed an entanglement monotone follows from the fact that marginal entropies $S(\rho_j)$ do not increase under local operations and classical communication. The above definition is based on, but differs from, the entanglement measure given in Ref. [21] in that Eq. (5) does not reduce to entropy of entanglement for bipartite states, but to twice that value. In general, Eq. (5) yields $N \log D$, and not $\log D$, for a maximally entangled N -partite state. This is convenient for our present need.

The advantage of definition Eq. (5) is that it suggests a direct extension to resolvable entanglement for QM_μ . It is well known that any bipartite system can be Schmidt decomposed into a state summed over a single index [22]. Precisely if the system is separable, its Schmidt number (number of terms in the Schmidt decomposition, which is not larger than the dimension of the smaller of its two constituent sub-systems) is 1. The Schmidt coefficients can be obtained directly as the eigenvalues of the reduced density operator of either sub-system. Let $(\lambda_+)_y$ denote the second largest eigenvalue of ρ_y (or $\rho_{\bar{y}}$). If for a particular bi-partition (y, \bar{y}) , we find that $(\lambda_+)_y < 2^{-\mu/2}$, then the entanglement between the sets y and \bar{y} is deemed unresolvable. The two parts are then effectively separable. On the other hand, if $(\lambda_+)_y \geq 2^{-\mu/2}$, then the entanglement is deemed resolvable, and the parts y and \bar{y} are said to be resolvably entangled to each other.

Thus, we define μ -bit resolvable (or μ -resolvable) entanglement by:

$$\xi_\mu^{(N)} = \begin{cases} \sum_{j=1}^N S(\rho_j) & \text{if } (\lambda_+)_y \geq 2^{-\mu/2} \ \forall \ y \in \mathcal{T}. \\ 0 & \text{otherwise.} \end{cases} \quad (6)$$

Two systems that are not μ -resolvably entangled are said to be μ -separable, that is, separable at μ -bit accuracy. The discretized evolution U_μ corresponding to a unitary operation U is resolvably entangling if, acting on systems that are effectively separable, it can produce μ -resolvable entanglement.

B. Finite quantum parallelism

A consequence of finite μ is that, if we assume that QM_μ is consistent and a reasonable approximation of QM, then there is an upper bound, D_{\max} , to the dimension of the Hilbert space of a *monolithic* system. Such a system is defined as either a single fundamental object (whatever it may be), or an entangled composite of two or more such objects. That is, by definition, a monolithic object is not composed of two or more μ -separable objects (fundamental or otherwise). Suppose an isolated, monolithic system exists in a state $|\psi\rangle = \sum_{j=0}^{D-1} a_j |j\rangle$, in some basis $\{|j\rangle\}$. If dimension $D > 2^\mu$, then it follows that there exists at least one j in this basis such that $|a_j| < 2^{-\mu/2}$, and therefore cannot be *resolved*, even to its most significant digit. This holds true for any other basis. Therefore, the *effective* dimension D_{eff} of an isolated, monolithic system in QM_μ must satisfy:

$$D_{\text{eff}} \leq 2^\mu, \quad (7)$$

or, equivalently, $\mathcal{A} \leq 2^\mu \mu$ bits. Any coherently evolving state in \mathcal{H}_μ is therefore *algorithmically bounded*.

Physically, this means that the coherent evolution of any physical system can proceed along at most 2^μ parallel superpositional pathways (terms in a coherent superposition). Thus, infinite parallelism in a continuum \mathcal{H} is replaced by *finite parallelism* in \mathcal{H}_μ . Let us consider a monolithic system accessing $D < 2^\mu$, which, by an abrupt absorption of energy, would have required access to $D' > 2^\mu$ in QM. CMQM posits that in such a situation, the non-resolvability of some amplitudes leads to loss of information from the system's state. The subsequent evolution of the object's state depends on whether this lost information is *significant* or not, in the sense clarified below.

Further, consider a system with finite average energy $\langle E \rangle = \sum_j p_j E_j$, where $p_j \geq 2^{-\mu}$ and $|\sum_j p_j - 1| \sim O(2^{-\mu})$. It follows that all E_j 's ($1 \leq j \leq 2^\mu$) are finite. Therefore, along each computational pathway, any such system evolves at a finite speed given by $2^{\mu/2} E_j / \hbar$ ops. As a result, finite parallelism in the discrete Hilbert space entails that any coherently evolving system corresponds to a finite rate of logical operations along finitely many computational pathways in the sub-physical matrix. In this sense, the quantum universe is not *computationally dense*, both in time and in \mathcal{H} . Formally, the sub-physical matrix 'simulates' the physical evolution (of systems) of the physical universe in a truncated basis of dimension at most 2^μ and at finite speed. In view of Eq. (2), to any operation \mathcal{U}_μ in \mathcal{H}_μ is associated an algorithmic information of $D^2 \mu$ bits, i.e., the dimension of U in \mathcal{H} times μ . In the Heisenberg picture, we have that \mathcal{U}_μ is updated at a rate of about $2^{\mu/2} D^2 \bar{E} / \hbar$ ops, where $D \leq 2^\mu$.

Even for modest values of μ , such as say 100 bits, low dimensional systems can hardly be distinguished from the continuum case. Further, if the Hilbert space dimension of the universe ($= \exp(S/k_B)$, where S is entropy of the universe and k_B , Boltzmann's constant) were much less than 2^μ , then the effect of finiteness of μ is not expected to show up easily at any scale. Yet, clearly, even familiar systems are conventionally considered as infinite dimensional, e.g., a coherent state of light, $|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} (\alpha^n / \sqrt{n!}) |n\rangle$. CMQM implies that the n_r^{th} and later terms in the summand will be unresolved, where n_r ($\leq 2^\mu$) is the smallest n such that $2^{-\mu/2} > e^{-|\alpha|^2/2} (|\alpha^n| / \sqrt{n!})$. For example, a coherent pulse with $\alpha = 2$ gives $n_r = 45$, so that the effective dimension $D_{\text{eff}} \leq 45$. By the remainder theorem for the Taylor series for $f(x) = e^{-\alpha^2} \exp(x = \alpha^2)$, we find that the truncated series sums up to not more than e^{-65} . On the other hand, the average probability for the included terms is about $1/45 \gg e^{-65}$. Since the total 'loss of probability' through non-resolution (the truncation error) is much smaller than the average probability in each included superposition term, the unresolvability of the amplitudes for $n > n_r$ is deemed *insignificant* and can be ignored. It is understood that what is usually taken to be the state $|\alpha\rangle$ is in fact physically realized as $|\alpha_\mu\rangle \equiv \sum_{n=0}^{\leq n_r} (e^{-|\alpha|^2/2} \alpha^n / \sqrt{n!}) |n\rangle$, where $[y]$ is the μ -bit rounded amplitude of the system. Clearly, it is difficult to practically distinguish $|\alpha_\mu\rangle$ from $|\alpha\rangle$. However, there are situations where the loss of amplitude information can be significant, and can thus not be ignored, as discussed below.

C. Computational instability

A set of objects may each satisfy Eq. (7), and yet, by interacting via interactions that are algorithmically bounded, they may still give rise to a monolithic system that violates Eq. (7) and can thus result in unresolvable amplitudes. A simple example is of N effectively D -dimensional particles such that $D \ll 2^\mu$, but $D^N > 2^\mu$. Of course, the latter fact by itself does not imply that there is *significant* unresolvability of amplitudes.

If these N particles are separable, then the unresolvability is statistically insignificant because each monolithic (separable) unit within the system satisfies Eq. (7). But if all the particles become strongly entangled, from Eq. (4) it follows that all the $\sim D^{N+1}$ nonlocal invariants differ significantly from their separable values and hence nearly all amplitudes are significant. The loss of probability that would result through non-resolution will thus be substantial. Therefore, the strong interaction regime, in which $\xi_\mu^{(N)} \rightarrow N \log D$, entails significant unresolvability.

We can describe the passage from the resolvable to the significantly unresolvable situation using parameter χ , defined as the largest entanglement $\xi_\mu^{(j)}$ ($j \leq N$) for any subset of the N particles in question:

$$\chi \equiv \max_y \xi_\mu^{(|y|)} \quad \forall \quad y \in (\mathcal{T} \cup T). \quad (8)$$

The condition for the unresolvability to be statistically *insignificant* is therefore:

$$\chi \ll \mu. \quad (9)$$

In the fully separable regime, where each of the N particles forms a ‘separable island’ in a pure state, $\chi = 0$, satisfying Eq. (9). But in the strong interaction regime, which results in near-maximal entanglement and nearly all amplitudes are statistically significant, $\chi \rightarrow N \log D > \mu$, and Eq. (9) fails. At this critical point, the system becomes *computationally unstable*, in the sense that the sub-physical simulation of the system becomes potentially very lossy. A physical system is computationally stable only if Eq. (9) is satisfied.

A primary element of CMQM is that ‘collapse of the wavefunction’ or ‘reduction of the state vector’ is the error preventive response of the sub-physical matrix to computational instability. Wavefunction collapse is modelled as a highly discontinuous transition during which χ abruptly shifts from about μ to 0 or a value much smaller than μ , as the system is projected from a state of immense entanglement to a product state (though the latter may not be separable in terms of the most basic degrees of freedom).

This is postulated to occur through the following two-step random procedure. As a system of N objects becomes computationally unstable, any one of the objects, which we call the trigger, collapses by a random projection into a basis whose selection is clarified below. Its state vector thus products out from the remaining objects’. Simultaneously, the latter are projected into the state correlated with that of object k . The full collapse therefore consists of the initial trigger-collapse, and the subsequent correlated collapse. The choice of the post-collapse state is assumed to be random subject to the Born probability rule. For the case of Eq. (1), the final state can be any $|j\rangle|m_j\rangle|E_j\rangle$ with probability $|c_j|^2$. The present model does not explain the origin of this randomness, which is taken to be a fundamental feature.

There are several features of the model that are novel to the issue of QMP. First is the feature that wavefunction collapse is related to the finiteness of memory and computational capacity available to the universe. Physically, this corresponds to the discreteness of Hilbert space. Our model suggests that wavefunction collapse is an algorithmic rather than dynamical phenomenon. By the term algorithmic, as against dynamical, is meant that wavefunction collapse corresponds rather to discrete computations and a re-setting of memory registers in the sub-physical matrix, than to a conventional Hamiltonian-driven evolution. We venture to suggest that computational instability and wavefunction collapse in CMQM are analogous to *segmentation fault* [23] and *crash* [24] of ordinary, digital computer programs. As in a crash, wavefunction collapse is characterized by loss of information, corresponding to the destruction of nonlocal correlations.

A further possibility is that wavefunction can be interpreted physically as an abstract phase transition, with χ as the order parameter. The phase transition is characterized by symmetry breaking as the state vector jumps from a space of larger symmetry of the highly entangled, computationally unstable state to a one of lower symmetry.

D. Algorithmic minimization

The above analysis did not address the preferred basis problem, QMP2. The latter’s resolution in CMQM relies on the fact that there is a unique basis that minimizes the average algorithmic information of the post-collapse state. For the N -particle computationally unstable system, let the k th particle be the trigger object projected into some basis $\beta = \{\beta_j\}$. Let the corresponding ensemble of states of the remaining objects obtained by correlated projection be $\phi = \{\phi_j\}$. As a simple example, consider the (unnormalized) maximally entangled state of three qubits, $|\psi\rangle = |000\rangle + |111\rangle$, with $k = 0$. For $\beta = \{|0\rangle, |1\rangle\}$, we find $\phi = \{|00\rangle, |11\rangle\}$, with $\bar{\chi} = 0$; for $\beta = \{|\pm\rangle\}$, we find $\phi = \{|00\rangle \pm |11\rangle\}$, where $|\pm\rangle = |0\rangle \pm |1\rangle$, with $\bar{\chi} = 2$. If we require that the average χ of ϕ should be minimized for the post-collapse ensemble, it is easily seen that for the state $|\psi\rangle$, the first basis is preferred.

More generally, suppose a computationally unstable system is given by the state $|\psi\rangle = \sum_j c_j |j_0\rangle |j_1\rangle \cdots |j_{N-1}\rangle$. Without loss of generality, setting the trigger coordinate $k = 0$, suppose that this particle collapses in a basis $\{|m\rangle\}$ other than $\{|j_0\rangle\}$, given by $|j_0\rangle = \sum_m \alpha_{jm} |m\rangle$. A projection of the trigger into an eigenstate in the $\{|m\rangle\}$ basis

leaves the remaining particles in the entangled state (apart from a normalization factor) $\sum_j c_j \alpha_{jm} |j_1\rangle \cdots |j_{N-1}\rangle$ and a corresponding $\bar{\chi} \approx (N-1) \log D$. On the other hand, if the trigger basis is $\{|j_0\rangle\}$, we obtain $\bar{\chi} = 0$. It is obvious that this holds true for any k . We thus see that the bases of objects in which their entangled state can be expanded through a single index minimizes the average algorithmic information of the resulting ensemble. Formally, this is equivalent to measurement in the basis $\{\hat{P}_j = (|j_0\rangle\langle j_0|) \otimes (|j_1\rangle\langle j_1|) \otimes \cdots \otimes (|j_{N-1}\rangle\langle j_{N-1}|)\}$, which is unique to the state $|\psi\rangle$ as the basis that permits a single index expansion of the latter. No matter what the trigger coordinate, the result is a projection of $|\psi\rangle$ in the basis $\{\hat{P}_j\}$.

Generalized to any system, this forms the *algorithmic minimization* principle of CMQM. Formally, consider ensemble $\phi = \{\phi_j\}$ correlated with the projection of the trigger object in a computationally unstable system, in basis $\beta = \{\beta_j\}$. We denote the average resulting entanglement by $\bar{\chi}(\beta) \equiv \sum_j \chi(\phi_j)$. The *algorithmically minimal basis* $\gamma = \{\gamma_j\}$ is defined as the one that minimizes $\bar{\chi}$:

$$\bar{\chi}(\gamma) = \min_{\beta} \bar{\chi}(\beta). \quad (10)$$

For states of the form (1), which are quite general for measurements in the von Neumann measurement paradigm [25], the basis $\{|j\rangle, |m_j\rangle, |E_j\rangle\}$ uniquely satisfies Eq. (10).

Thus, apart from computational instability and probabilistic collapse, the third main element of CMQM is that the final (possibly random) state following collapse is chosen from the (algorithmically) minimal basis. This is a ‘reasonable’ response to the information overflow experienced during computational instability. Crucially, it helps resolve the preferred basis problem (QMP2) as it singles out a specific basis in which the system is actualized or macro-objectified. The particular element in the minimal basis which the system collapses to is randomly chosen, subject to the condition that probability $p_j = \text{Tr}(\hat{P}_j |\psi\rangle\langle\psi|)$. If we restrict attention to a subsystem in a monolithic system, the subsystem’s evolution is given by a completely positive map on density operators [22].

As the particles in the system remain dynamically interacting, a collapse is followed by an episode of μ -unitary evolution, during which interaction re-entangles the system, making it computationally unstable again. This is succeeded by a collapse, and so on. The perpetual cycle of alternating collapses and μ -unitary evolutions gives rise to a classical behaviour. To see this, note that in the continuous limit, assuming Markovian (time-local) conditions, the collapse of any (open) sub-system can be represented by the action of Lindblad operators. This results in an evolution of the density operator described by a Lindblad-type master equation [2, 3], which often suffices to explain the emergence of macro-scale classical behaviour. In particular, this can also elucidate why position often emerges as the preferred basis.

The connection of our model to decoherence is worth stressing. Notice that the form Eq. (1), in which the states $\{|E_j\rangle\}$ are practically orthogonal, results from decoherence. As a consequence, the system-apparatus complex loses all coherence. Non-selectively, the density operator of this subsystem is the same as would be obtained by projective measurements in the $\{|E_j\rangle\}$ basis. Therefore, statistically, CMQM is indistinguishable from decoherence. Again, in the more general context of evolution of macroscopic open systems, we noted that CMQM yields a Lindbladian evolution. Here too, the effect of the CMQM scenario is identical to that due to decoherence. However, CMQM has the added feature of being able to explain the appearance of specific outcomes, and can thus serve as a complement to decoherence that terminates the measurement process.

CMQM implies that the parameter μ determines the Heisenberg cut, the mesoscopic threshold presumably separating the quantum microcosm from the classical macrocosm. If μ were larger, then computational instability would be attained later than earlier, and so quantum superpositions would be seen at larger scales. According to CMQM, the classicality of the macro-world is due to the ‘accident’ that μ is too small in comparison with the degrees of freedom of the universe. Following Ref. [16], suppose that the entropy S of the universe is 10^{120} . The corresponding dimension is $D_{\text{univ}} = \exp(S/k_B)$. According to CMQM, if $2^\mu \gg D_{\text{univ}}$, then even the macroscopic world would be quantum rather than classical. Conversely, the fact that the macro-world is classical therefore implies that $\mu < 10^{143}$.

In fact, μ is probably much smaller. We suggest that experiments of the type studied in Refs. [26, 27] (and references cited therein) can possibly help determine the value of μ by identifying the mesoscopic scale at which quantum behaviour transitions to classical. However, it should be noted that these experiments cannot be directly used for our purpose. They rely on identifying quantum behaviour interferometrically and thus do not distinguish between an actual collapse and the mere loss of quantum coherence, i.e., between selective and non-selective measurements. As a result, they are insensitive to the difference between the effect of decoherence and that of decoherence terminated by a CMQM collapse. We believe that modifications of such experiments can nevertheless be used to quantify μ .

III. LINKS TO OTHER FUNDAMENTAL PROBLEMS

We note two consequences of CMQM. First, there is an asymptotic limit to the power of quantum computation. Consider, for example, Shor's algorithm for prime factorization [28]. To factorize a number n , we choose a number a that is co-prime to n and produce the entanglement $n^{-1/2} \sum_j |j\rangle|0\rangle \rightarrow n^{-1/2} \sum_j |j\rangle|f(j)\rangle$, by way of determining the period of the function $f(j) = a^j \bmod n$. However, according to Eq. (9), we should have $n \leq 2^\mu$, which is thus the largest number that can be factorized using this algorithm. A quantum computer that attempts to access higher dimensions will collapse, losing coherence. Note that according to CMQM classicality of the macro-world itself is due to interactions leading physical objects to attempt to access larger than 2^μ dimensions. The physical universe can thus be regarded as a quantum computer that is dimensionally too rich, and turning classical as a consequence. Interestingly, a different approach to discreteness of Hilbert space limiting the power of quantum computers is reported in Ref. [19].

Second, we remark on some connections to quantum gravity. General Relativity (GR) is well-tested at macroscopic scales. Yet, when extrapolated to very smaller scales, it encounters inconsistencies in the form of singularities. For this, among other reasons, one suspects that GR is not universally valid, but that at sufficiently small scales, a theory of quantum gravity would be required. CMQM can be motivated along similar lines, by arguing that the universal validity of quantum mechanics at all amplitude scales would imply macroscopic superpositions, contrary to observations, and that this calls for new physics at very small (but significant) amplitudes. The loop quantum gravity [29] approach predicts that spacetime does not form a continuum, but is discrete. This guarantees avoidance of classical singularities as well as the high frequency divergences of quantum field theory. Similarly, CMQM requires the discreteness of the space of states, which guarantees avoidance of macroscopic superpositions by precluding arbitrarily massive quantum parallelism.

To manifest the possible granularity of space we require very high energies in order to probe Planck length phenomena. Analogously CMQM implies that to manifest the granularity of state space, massive superposition (entanglement) is needed. Yet, a dramatic contrast between these two kinds of granularity is that whereas the former requires exotic conditions (Planck scale energies) to be manifested, the latter is almost ubiquitous and inescapable: in the classicality of the world we ordinarily see around us.

We claim that finite μ implies discreteness of spacetime, an idea quite familiar in certain approaches to quantum gravity, notably loop quantum gravity, as noted above. At a given time, let us consider a cubic region of space, R , of length L , that is sufficiently small that the wavefunction ψ of a particle hardly varies over it. To begin with consider space as divided into finite number of cells of size Δx . The number of degrees of freedom in R is $N = (L/\Delta x)^3$. If the total probability of finding the particle in R is $p \leq 1$, then the average amplitude α in this region satisfies $\alpha \leq \sqrt{p/N} = \sqrt{p}(\Delta x/L)^{3/2}$. As $\Delta x \rightarrow 0$, we have $\alpha \rightarrow 0$. In particular, if $N > 2^\mu$, then $\alpha < 2^{-\mu/2}$ and the amplitudes in the lattice in the region R are unresolvable. This argument can be applied to every other similarly chosen region R where the particle has some resolvable probability to be found. We therefore require that $\Delta x > L/2^{\mu/3}$. The discreteness in spacetime need not imply regularity. Space need not be a lattice, but might be given by a probabilistic distribution consistent with the demands of Relativity theory. The latter condition will also imply a corresponding discreteness in time, which by the way is also suggested by Eq. (3). Conversely, one can also motivate a discreteness of Hilbert space, starting from discreteness of space [19].

In conclusion, we believe our work opens a possible approach to realize Wheeler's phrase "it from bit", namely, that physical reality derives its existence from a deeper information theoretic layer [11], which we have called the sub-physical universe/matrix. To quote Wheeler, "'It from bit' symbolises the idea that every item of the physical world has at bottom - at a very deep bottom, in most instances - an immaterial source and explanation; that which we call reality arises in the last analysis from the posing of yes-no questions and the registering of equipment-evoked responses; in short, that things physical are information-theoretic in origin.' Perhaps the most surprising aspect of our proposed model is that the possible profound connection between physics on the one hand, and information theory and computer science on the other, that it suggests finds a very commonplace manifestation- in the classicality of the familiar macroscopic world.

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